Physics 371: Problem Set 1

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Due Thursday 15 April, 1:30 p.m.

In class, we discussed the flatness problem: To get an almost flat Universe today, we need an extremely flat Universe in the past. We showed that an inflationary period gives a natural solution to this problem. There is another problem that inflation helps to solve: The so-called horizon problem, related to the fact that the Cosmic Microwave Backround radiation of 2.7 K observed today seems to have originated from regions of space that were not in causal contact at the time of its creation. The puzzling feature to explain is the high degree of homogeneity and isotropy of the radiation background.

1. In cosmology, one can introduce the concept of "horizons": The past light cone of an event can intersect the big bang at a finite distance, so there will be particles whose worldlines do not intersect that light cone.

1. Consider two particles at the epoch of recombination, when the electrons, protons and neutrons form neutral atoms. At that point, we believe, the microwave background radiation was formed. Under our present understanding of the evolution of the Universe, this should have happened at a redshift of about $z = 1000$. The scale factor was, then about 1000 times smaller than what it is today. Imagine that at that time two particles are just outside each other's horizon; that is, the past light cones just touch at the big bang. Imagine further that the universe has been flat and matter-dominated for its whole history (not true, but imagine it). What is the presently observed angular separation of these two points on the sky?

   Hint: The horizon can be defined from the Eq. describing the propagation of light $(ds^2 = 0, dt = R(t)dr)$. Then, the horizon distance is just

   $$R(t) \int_0^{r_H} dr = R(t) \int_0^t \frac{dt'}{R(t')} \tag{1}$$

2. Now imagine a universe which is flat and has been matter dominated ever since some redshift z_*, but before that it was vacuum-dominated. That is, imagine that that all of the energy density in the universe was vacuum $(p = -\rho)$ up to z_*, then suddenly turned into matter $(p = 0)$ at a phase transition (nothing wrong with that). Show

that, if you extrapolate the inflationary period back in time until the "origin of the Big Bang", $R(t) \to 0$, the past light cones of any two points will intersect in the past.

2. Plot the scale factor vs. time for FRW universes with the following choices for the parameters $(\Omega_{\mathrm{M},0}, \Omega_{\mathrm{R},0}, \Omega_{\Lambda,0})$: (1, 0, 0); (0, 1, 0); (0.1, 0, 0); (5, 0, 0); and (0.3, 0, 0.7). Taking the case (0.3,0,0.7), close to what it is today: How long will it take for the density of matter to be less than one percent of the critical density ($\Omega_M = 0.01$ and $\Omega_\Lambda = 0.99$) ? Extra credit: You can solve the Friedmann equation numerically, and plot your curves on the same graph, with the same value of the Hubble constant today.